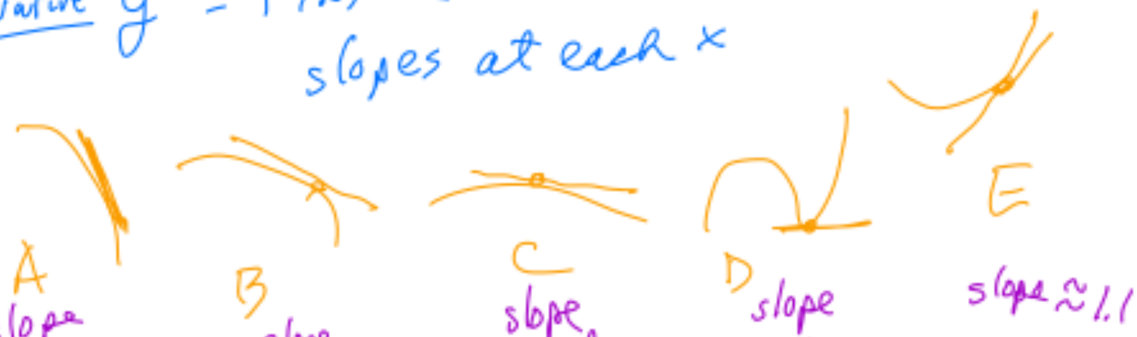
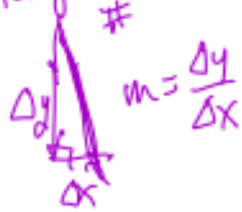
Graphs and Derivatives.

$y = f(x)$ ← height (vertical position) of graph at each x

can look at the graph.

derivative $y' = f'(x)$ ← tells us how the graphs slopes at each x

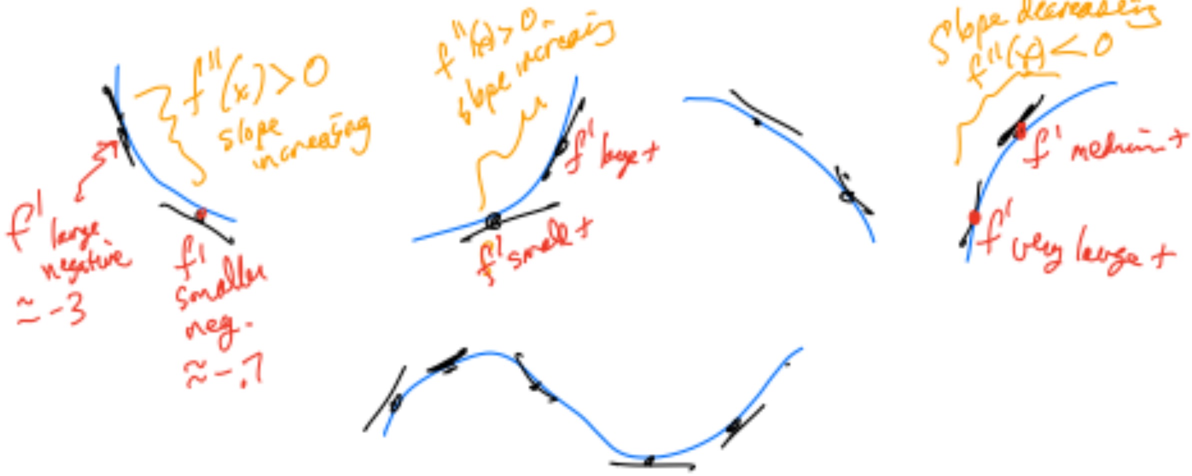




Second derivative $y'' = f''(x) = (f'(x))'$ = rate of change of slopes.

$f''(x) > 0 \Rightarrow$ slopes are going up in value (increasing)

$f''(x) < 0 \Rightarrow$ slopes are decreasing



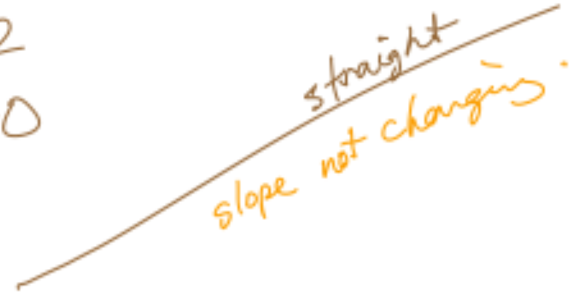
$f''(x) > 0 \Leftrightarrow$ increasing slope $\Leftrightarrow$ concave up (looks like a cup)

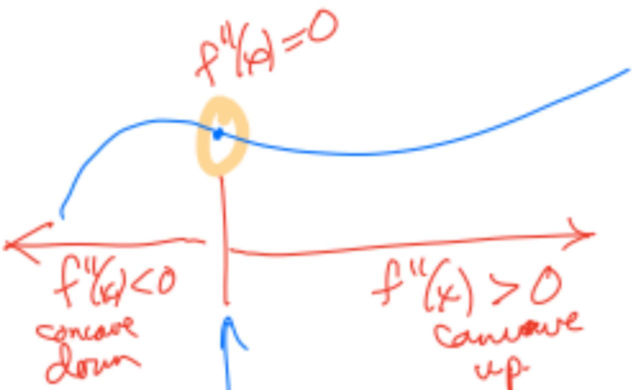
$f''(x) < 0 \Leftrightarrow$ decreasing slope $\Leftrightarrow$ concave down (looks like a frown)

$f''(x) = 0$ $\quad y = 2x - 4$

$y' = 2$

$y'' = 0$





An <u>inflection point</u> is a point where the graph switches from concave up to concave down. (or cc down to cc up.)

(Note: $f''(x)=0$ does not mean you have an inflection point.)

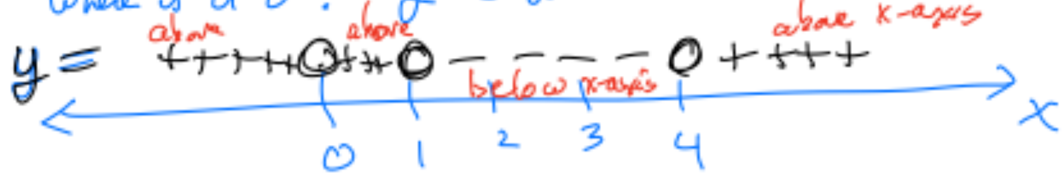
Example Analyze the graph of

$$y = x^2(x-1)(x-4).$$

We will look at where y, y', and y'' are positive, negative, or zero.

function $y = x^2(x-1)(x-4)$.

where is it 0? $y=0$ when $x = 0, 1,$ or 4.



$$y' = 2x(x-1)(x-4) + x^2(x-4) + x^2(x-1)$$

Where is this zero? Simplify & factor first.

$$y' = x\left[(2)(x^2-5x+4) + x^2-4x + x^2-x\right]$$

$$= x\left[2x^2-10x+8+x^2-4x+x^2-x\right]$$

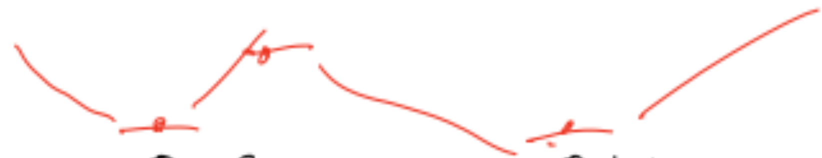
$$= x\left[4x^2-15x+8\right]$$

$(2x- \quad)(2x- \quad)$
$(4x- \quad)(x- \quad)$ doesn't work.

quadratic formula $x = \dfrac{15 \pm \sqrt{225-128}}{8}$

$$x = \frac{15 \pm \sqrt{97}}{8}$$

$$y' = 4\underset{0}{x}\Big(x - \underset{.6}{\frac{15-\sqrt{97}}{8}}\Big)\Big(x - \underset{3.1}{\Big(\frac{15+\sqrt{97}}{8}\Big)}\Big)$$



$y' = - - - - 0 + + + 0 - - - - - 0 + + + +$

$0 \quad .6 \quad 1 \quad 2 \quad 3 \quad 3.1$

$y'' = ?$

$y' = x[4x^2 - 15x + 8] = 4x^3 - 15x^2 + 8x$

$y'' = 12x^2 - 30x + 8 = 2(6x^2 - 15x + 4)$

where is this zero?

$= 2(3x - 1)(2x - 4)$ X

$2(6x - 2)(x - 2)$ X

quadratic formula roots

$y'' = 12(x - \quad)(x - \quad)$

$x = \dfrac{15 \pm \sqrt{225 - 96}}{12}$

$y'' = 12\left(x - \dfrac{15 - \sqrt{129}}{12}\right)\left(x - \dfrac{15 + \sqrt{129}}{12}\right) \quad = \dfrac{15 \pm \sqrt{129}}{12}$

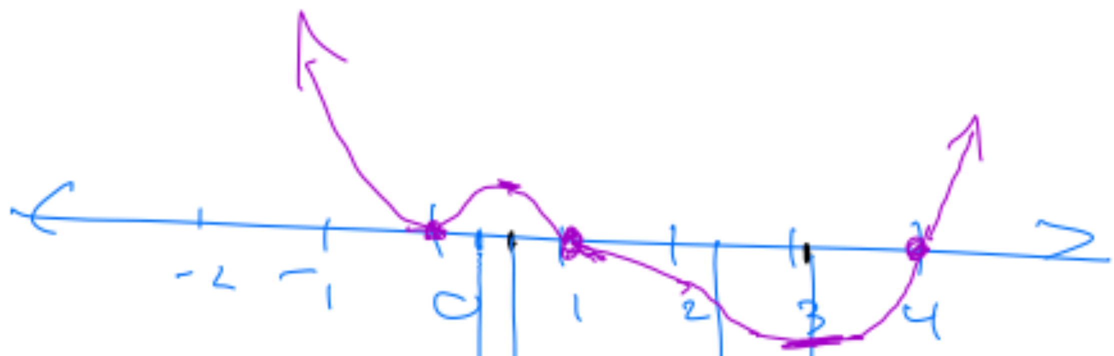
$.3 \qquad 2.3$

CC up   CC down   CC up

$y'' = + + + + + + 0 - - - - - - 0 + + + +$

$0 \quad .3 \quad 1 \quad 2 \quad 2.3$

$y =$ 



$y =$ + + + + 0 + + 0 − − − − 0 + + +

$y' =$ − − − − − − − 0 + 0 − − − − 0 + + + +

$y'' =$ + + + + 0 − − − 0 + + + + +